Endogenously sticky price

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Abstract

In this paper

1 Introduction

The seminal work by Gali and Monacelli (2005) has been the foundation for monetary policy analysis in much of the recent research. A key assumption in this framework is exogenously sticky price. However, a recent empirical analysis by Singh, Suda, and Zervou (2022) revealed the different sectors respond differently to monetary policy.

In this paper, I explore the theoretical reason behind the observed heterogeneous response. The hypothesis is that different sectors have different price stickiness that arises from inventory and shelf life of the product.

Relation to the literature

This work builds upon the existing literature on inventory management. Some of the early works in this area were done in Arrow et al. (1951), and Bellman et al. (1955). They derive optimal inventory policy under various conditions, however, assuming demand function to be exogenously given. Scarf (1960), and Iglehart (1963) explore the optimality of (s,S) inventory policy in a dynamic set up.

The (s,S) policy of inventory management is one of the widely studied model by economists. Iglehart (1963) study convergence of economies aggregate to frictionless counterpart. Khan and Thomas (2007) incorporate (s,S) into a business cycle model to explain cyclical variability of inventory investment.

The inventory management problem has also been extensively explored in operations research literature. This paper is close to continuous time models as described in Benkherouf and Gilding (2021a), Benkherouf and Gilding (2021b), and Alfares and Ghaithan (2016). The contribution of this paper is to study the inventory decision with non linear price dependent demand rate.

2 A model with sticky price

Household

There are a continuum of of mass 1 household that maximizes their lifetime utility over consumption and leisure. The expected lifetime utility is given by:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(C_t, N_t), \tag{1}$$

where C_t is a consumption index given by

$$C_t = \left(\int_0^1 C_t(i)^{1-\frac{1}{\varepsilon}} \right)^{\frac{\varepsilon}{\varepsilon-1}}. \tag{2}$$

I assume the following functional form for $U(C_t, N_t)$,

$$U(C_t, N_t) = \left(\frac{C_t^{1-\sigma}}{1-\sigma} - \psi \frac{N_t^{1+\phi}}{1+\phi} \right), \tag{3}$$

and β is the discount factor, $\sigma > 0$ is that elasticity of intertemporal substitution, and $\phi > 0$ is the Frisch elasticity of labor supply. The household's optimization problem is subject to the budget constraint given by:

$$P_t C_t + Q_t B_t = W_t N_t + B_{t-1} + P_t \mathcal{F}_t \tag{4}$$

where B_t is the number of shares of government bond, Q_t is the price of government bond, and $\mathcal{F}_t$ is the profits of the firm.

Let $\Pi_t = \dfrac{P_t}{P_{t-1}}$ denote the inflation rate. The households' choice can be characterized by the following first order conditions

$$-\frac{U_{n,t}}{U_{c,t}} = \frac{W_t}{P_t} = \psi C_t^{\sigma} N_t^{\phi}, \tag{5}$$

$$Q_t = \beta \mathbb{E}_t \left(\frac{U_{c,t+1}}{U_{c,t}} \frac{P_t}{P_{t+1}} \right). \tag{6}$$

The final good producer

The final good (Y_t) is produced using the intermediate goods Y_{it} using the following technology:

$$Y_t = \left(\int_0^1 Y_{it}^{\frac{\varepsilon-1}{\varepsilon}} di \right)^{\frac{\varepsilon}{\varepsilon-1}}, \tag{7}$$

where ε is the elasticity of substitution.

Final good producers are perfectly competitive in the input and output market. They maximize profits subject to the production function (7). Therefore the input demand function is given by:

$$Y_{it} = \left(\frac{P_{it}}{P_t} \right)^{-\varepsilon} Y_t \quad \forall i \tag{8}$$

and the price index is

$$P_t = \left(\int_0^1 P_{it}^{1-\varepsilon} \right)^{\frac{1}{1-\varepsilon}}. \tag{9}$$

Intermediate good producers

Each intermediate firm produces a differentiated good using only labor as input and a linear production technology:

$$Y_{it} = A_t N_{it},$$

where ℓ_{it} is the labor hired by the firm i at time t. The productivity $a_t = \log(A_t)$ follows the following AR(1) process:

$$a_{t+1} = \rho a_t + \epsilon_{at}. \tag{10}$$

The intermediate good producers are monopolistically competitive in output market, and they reoptimize their prices P_{it} at time t with probability $1 - \theta$, with $0 < \theta < 1$. A firm which does not reoptimize the price continue with their old prices. A firm that reoptimizes its price at time t, maximizes the current real value of profits generated while the price remains effective:

$$\max_{X_{it}} \mathbb{E}_t \sum_{\tau=t}^{\infty} \theta^{(\tau-t)} Q_{t,\tau} \left(\frac{X_{it}}{P_\tau} Y_{i\tau|t} - MC_\tau Y_{i\tau|t} \right)$$

$$\text{s.t. } Y_{i\tau|t} = \left(\frac{X_{it}}{P_\tau}\right)^{-\varepsilon} Y_\tau,$$

where,$Q_{t,\tau}$ is the stochastic discount factor

$$Q_{t,\tau} = \beta^{\tau-t}\mathbb{E}_t\left[\left(\frac{C_\tau}{C_t}\right)^{-\sigma}\frac{P_t}{P_\tau}\right]$$

MC_τ is the real marginal cost of the intermediate good producers which is same across all firms

$$MC_t = \frac{W_t}{P_t A_t}.$$

Now, the first order condition of the firm with respect to p_{it} is

$$\mathbb{E}_t\sum_{\tau=t}^{\infty}\theta^{(\tau-t)}Q_{t,\tau}(1-\varepsilon)\left(\frac{X_{it}}{P_\tau}\right)^{-\varepsilon}\frac{Y_\tau}{P_\tau} = -\mathbb{E}_t\sum_{\tau=t}^{\infty}\theta^{(\tau-t)}Q_{t,\tau}MC_\tau\varepsilon\left(\frac{X_{it}}{P_\tau}\right)^{-\varepsilon-1}\frac{Y_\tau}{P_\tau}$$

$$\Rightarrow X_{it}\mathbb{E}_t\sum_{\tau=t}^{\infty}\theta^{(\tau-t)}Q_{t,\tau}\frac{Y_\tau}{P_\tau^{1-\varepsilon}} = \frac{\varepsilon}{\varepsilon-1}\mathbb{E}_t\sum_{\tau=t}^{\infty}\theta^{(\tau-t)}Q_{t,\tau}\frac{W_\tau}{A_\tau}\frac{Y_\tau}{P_\tau^{1-\varepsilon}}$$

New Keynesian Phillips Curve

The New Keynesian Phillips curve for this model can be derived by taking first order approximation around zero inflation steady state. Let the normalized price index in zero inflation steady state be 1. In addition, $Y_\tau = Y$, $Q_{\tau,t} = \beta^{\tau-t}$. Then the first order approximation around this steady state may be written as

$$x_{it} = (1-\beta\theta)\sum_{\tau=t}^{\infty}(\beta\theta)^{\tau-t}\mathbb{E}_t p_\tau^* \tag{11}$$

where, $p_\tau^* = \log(P_\tau^*)$, and $P_\tau^* = \dfrac{\varepsilon}{\varepsilon-1}\dfrac{W_\tau}{A_\tau}$ is the profit maximizing price of firm i in period τ.

Claim 1. *Around the flexible price equilibrium, the aggregate production function for this economy can be written as $Y_t = A_t N_t$ upto a first order approximation.*

Proof. content... ■

Claim 2. *The New Keynesian Phillips curve for the model with sticky price is given by*

$$\pi_t = \kappa \tilde{y}_t + \beta \mathbb{E}_t \pi_{t+1}, \tag{12}$$

where $\kappa = \dfrac{(1-\theta)(1-\beta\theta)(\sigma+\phi)}{\theta}$*, and* $\tilde{y}_t$ *denotes the output gap i.e. deviation of log output from the log of natural output level.*

Proof. We start with the optimal price setting equation (11)

$$x_{it} = (1-\beta\theta)\sum_{\tau=t}^{\infty}(\beta\theta)^{\tau-t}\mathbb{E}_t p_\tau^*$$

$$\begin{aligned}
x_{it} - p_{t-1} &= (1-\beta\theta)\sum_{\tau=t}^{\infty}(\beta\theta)^{\tau-t}\mathbb{E}_t(p_\tau^* - p_{t-1}) \\
&= (1-\beta\theta)(p_t^* - p_{t-1}) + \beta\theta\left[(1-\beta\theta)\sum_{\tau=t+1}^{\infty}(\beta\theta)^{\tau-(t+1)}\mathbb{E}_t(p_\tau^* - p_t)\right] \\
&+ \beta\theta(1-\beta\theta)\sum_{\tau=t+1}^{\infty}(\beta\theta)^{\tau-t-1}\mathbb{E}_t(p_t - p_{t-1}) \\
&= (1-\beta\theta)(p_t^* - p_{t-1}) + \beta\theta\mathbb{E}_t(x_{i,t+1} - p_t) + \beta\theta(p_t - p_{t-1}) \\
\Rightarrow \frac{\pi_t}{1-\theta} &= (1-\beta\theta)(p_t^* - p_{t-1}) + \beta\theta\mathbb{E}_t\frac{\pi_{t+1}}{1-\theta} + \beta\theta\pi_t \\
\Rightarrow \pi_t &= \frac{(1-\beta\theta)(1-\theta)}{1-\beta\theta(1-\theta)}(p_t^* - p_{t-1}) + \frac{\beta\theta}{1-\beta\theta(1-\theta)}\mathbb{E}_t\pi_{t+1} \\
\Rightarrow \pi_t &= \frac{(1-\beta\theta)(1-\theta)}{1-\beta\theta(1-\theta)}(p_t^* - p_t + \underbrace{p_t - p_{t-1}}_{=\pi_t}) + \frac{\beta\theta}{1-\beta\theta(1-\theta)}\mathbb{E}_t\pi_{t+1} \\
\Rightarrow \pi_t &= \frac{(1-\beta\theta)(1-\theta)}{\theta}(p_t^* - p_t) + \beta\mathbb{E}_t\pi_{t+1}
\end{aligned}$$

Next, we have to show that $p_t^* - p_t = (\sigma+\phi)(y_t - y_t^n)$, y_t^n is the log of natural output level under flexible pricing.

From the first order conditions of the households we have

$$
\begin{aligned}
\frac{W_t}{P_t} &= \psi C_t^{\sigma} N_t^{\phi} \\
\Rightarrow \frac{W_t}{P_t^*}\frac{P_t^*}{P_t} &= \psi C_t^{\sigma} N_t^{\phi} \\
\Rightarrow \frac{P_t^*}{P_t} &= \frac{\varepsilon\psi}{\varepsilon - 1}\frac{C_t^{\sigma} N_t^{\phi}}{A_t}
\end{aligned}
$$

Using Claim 1 and goods market clearing condition, we have,

$$
\Rightarrow \frac{P_t^*}{P_t} = \frac{\varepsilon\psi}{\varepsilon - 1}\frac{Y_t^{\sigma} Y_t^{\phi}}{A_t^{1+\phi}}.
$$

Under flexible price equilibrium, $P_t^* = P_t$, which results in $Y_t^{n(\sigma+\phi)} = \frac{\varepsilon - 1}{\varepsilon\psi} A_t^{1+\phi}$.

Hence, we get

$$
\frac{P_t^*}{P_t} = \frac{Y_t^{\sigma+\phi}}{Y_t^{n(\sigma+\phi)}}.
$$

Taking log and substituting it back into the expression for π_t, we get the NK Phillips Curve

$$
\pi_t = \kappa \tilde{y}_t + \beta \mathbb{E}_t \pi_{t+1},
$$

where $\kappa = \frac{(1-\theta)(1-\beta\theta)(\sigma+\phi)}{\theta}$. ■

Dynamic IS relation

Taking log of (6), we get the IS relation

$$
c_t = \mathbb{E}_t c_{t+1} - \frac{1}{\sigma}(i_t - \mathbb{E}_t \pi_{t+1} - \rho), \tag{13}
$$

where $i_t = -\log Q_t$, and $\rho = -\log \beta$. Applying market clearing condition we then get

$$
y_t = \mathbb{E}_t y_{t+1} - \frac{1}{\sigma}(i_t - \mathbb{E}_t \pi_{t+1} - \rho), \tag{14}
$$

The dynamic IS relation expressed in terms of output gap becomes

$$\tilde{y}_t = \mathbb{E}_t \tilde{y}_{t+1} - \frac{1}{\sigma}(i_t - \mathbb{E}_t \pi_{t+1} - r_t^n), \tag{15}$$

where r_t^n is the natural real rate of interest given by

$$r_t^n = \rho + + \frac{1+\phi}{\sigma+\phi}\mathbb{E}_t(a_{t+1} - a_t). \tag{16}$$

Interest rate rule

It is assumed that the central bank follows an interest rate rule of following form

$$i_t = \rho + \phi_\pi + \pi_t + \phi_y \tilde{y}_t + \nu_t, \tag{17}$$

where ν_t is an AR(1) process with zero unconditional mean. In addition, it is also assumed that ϕ_π, and ϕ_y are non negative constants chosen by the central bank.

$$\nu_t = \rho_\nu \nu_{t-1} + \epsilon_t^\nu. \tag{18}$$

Equilibrium dynamics

Combining equation (17), (12), and (15) we get the following linear rational expectation model

$$\begin{bmatrix} \nu_{t+1} \\ \mathbb{E}_t \pi_{t+1} \\ \mathbb{E}_t \tilde{y}_{t+1} \end{bmatrix} = \begin{bmatrix} \rho_\nu & 0 & 0 \\ 0 & \frac{1}{\beta} & -\frac{\kappa}{\beta} \\ 1 & \frac{1}{\sigma}\left(\phi_\pi - \frac{1}{\beta}\right) & 1 + \frac{1}{\sigma}\left(\phi_y - \frac{\kappa}{\beta}\right) \end{bmatrix} \begin{bmatrix} \nu_t \\ \pi_t \\ \tilde{y}_t \end{bmatrix} + \begin{bmatrix} \epsilon_{t+1}^\nu \\ 0 \\ 0 \end{bmatrix}; \tag{19}$$

with $\nu_0 = \epsilon_t^\nu$. Where $A_t = 1 \; \forall t$ has been assumed for simplicity.

3 A model with inventory

The description of households and final good firm is identical to the description in Section 2. However now we have two types of intermediate good firms - manufacturing and services. All firms use identical production technology which is linear in labor input. Manufacturing firms carry out their production activity in remote location and demand for manufactured good is fulfilled through retail units. On the other hand, the service firms produce their output at the location of demand.

Household

There are a continuum of of mass 1 household that maximizes their lifetime utility over consumption and leisure. The expected lifetime utility is given by:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\sigma}}{1-\sigma} - \psi \frac{N_t^{1+\phi}}{1+\phi} \right), \tag{20}$$

where β is the discount factor, $\sigma > 0$ is that elasticity of intertemporal substitution, and $\phi > 0$ is the Frisch elasticity of labor supply. The household's optimization problem is subject to the budget constraint given by:

$$P_t C_t + Q_t B_t = W_t N_t + B_{t-1} + P_t \mathcal{F}_t \tag{21}$$

where B_t is the number of shares of government bond, Q_t is the price of government bond, and $\mathcal{F}_t$ is the profits of the firm.

Let $\Pi_t = \dfrac{P_t}{P_{t-1}}$ denote the inflation rate. The households' choice can be characterized by the following first order conditions

$$-\frac{U_{n,t}}{U_{c,t}} = \frac{W_t}{P_t} = \psi C_t^{\sigma} N_t^{\phi}, \tag{22}$$

$$Q_t = \beta \mathbb{E}_t \left(\frac{U_{c,t+1}}{U_{c,t}} \frac{P_t}{P_{t+1}} \right). \tag{23}$$

Let the household allocate N_{mt} units of labor to the manufacturing sector, and N_{st} units of labor to the service sector, so that $N_{mt} + N_{st} = N_t$.

The final good producer

The final good (Y_t) is produced using the intermediate goods Y_{it} using the following technology:

$$Y_t = \left(\int_0^1 Y_{it}^{\frac{\varepsilon-1}{\varepsilon}} di \right)^{\frac{\varepsilon}{\varepsilon-1}}, \tag{24}$$

where ε is the elasticity of substitution.

Final good producers are perfectly competitive in the input and output market. They maximize profits subject to the production function (24). Therefore the input demand function is given by:

$$Y_{it} = \left(\frac{P_{it}}{P_t} \right)^{-\varepsilon} Y_t \quad \forall i, \tag{25}$$

and the price index is

$$P_t = \left(\int_0^1 P_{it}^{1-\varepsilon} \right)^{\frac{1}{1-\varepsilon}}. \tag{26}$$

Since the intermediate output comes from two sectors, the final good output can also be expressed as follows

$$Y_t = \left(\underbrace{\int_0^{\vartheta} Y_{it}^{\frac{\varepsilon-1}{\varepsilon}} di}_{\text{manufaturing output}} + \underbrace{\int_{\vartheta}^1 Y_{it}^{\frac{\varepsilon-1}{\varepsilon}} di}_{\text{service output}} \right)^{\frac{\varepsilon}{\varepsilon-1}}, \tag{27}$$

where ϑ is the fraction of manufacturing firms.

Since the output of individual sectors differ only in their index, therefore the quantity and price of each product within a sector will be same. That is the manufacturing output from the firm will be

$$Y_{imt} = \left(\frac{P_{mt}}{P_t} \right)^{-\varepsilon} Y_t \quad \forall i, \tag{28}$$

and the output of each intermediate good firm from the service sector will be

$$Y_{ist} = \left(\frac{P_{st}}{P_t} \right)^{-\varepsilon} Y_t \quad \forall i. \tag{29}$$

Finally the price index can be decomposed as

$$P_t = \left(\vartheta P_{mt}^{1-\varepsilon} + (1-\vartheta) P_{st}^{1-\varepsilon}\right)^{\frac{1}{1-\varepsilon}}. \tag{30}$$

Intermediate good firms

There are two types of intermediate goods - manufacture goods and service goods. Total mass of the intermediate good firms is 1, a fraction ϑ of them are manufacturing and rest are service firms. The manufacturing activity takes place at a location that is different from the location of demand. So these firms must rely on retail units to meet the demand from

Manufacturing unit

The description of manufacturing unit is as explained in Appendix A, with $\alpha = 1$. The firm's optimization problem is now characterized by

$$c_t A_{t-1} = W_{t-1}.$$

Because each manufacturing firm is identical, therefore the labor supply to manufacturing firm will be $N_{imt} = \frac{1}{\vartheta} N_{mt}$.

Retail unit

The behavior of retail unit is also identical to the description provided in Appendix A. It can be shown that for the linear production function assumed in this section the price of manufactured goods can be written as

$$P_{mt} = \frac{\varepsilon}{\varepsilon(1-\sigma) - 1} \frac{W_{t-1}}{A_{t-1}}. \tag{31}$$

Service sector

In this paper service sector does not maintain an inventory and therefore supply and demand should match every period. The absence of inventory then implies that the price of service sector goods will be

$$P_{st} = \frac{\varepsilon}{\varepsilon - 1} \frac{W_t}{A_t}. \tag{32}$$

Equilibrium

Definition 1. *A competitive equilibrium for the given setup with inventory is the sequence of price $\mathbb{P} =$; household choices $\mathbb{Z}_h =$; manufacturing unit choice $\mathbb{Z}_m =$; retail unit choices $\mathbb{Z}_r =$; and service sector firm choices $\mathbb{Z}_s =$, such that*

- *Given $\mathbb{P}$, households maximize their utility*
- *Given $\mathbb{P}$, manufacturing unit gets zero profit*
- *Given $\mathbb{P}$, retail units and service sector firms maximize their profits*
- *Prices $\mathbb{P}$ are such that markets clears*
    1. *Manufactured goods market*
    2. *Service goods market*
    3. *Bond market*

Phillips curve

In order to derive the Phillips curve, the first order approximation around zero inflation steady state is taken.

Zero inflation steady state

In this section the expressions from variables like output, employment, and price index are derived in zero inflation steady state.

Definition 2. *A zero inflation steady state is the economy in which the prices of each sector remains constant.*

The price of each sectoral good are given by

$$P_m = \frac{\varepsilon}{\varepsilon(1-\sigma)-1}\frac{W}{A}; \tag{33}$$

$$P_s = \frac{\varepsilon}{\varepsilon-1}\frac{W}{A}. \tag{34}$$

From here, the price index may be written as

$$P^{1-\varepsilon} = \vartheta P_m^{1-\varepsilon} + (1-\vartheta)P_s^{1-\varepsilon}. \tag{35}$$

Given the expression for prices we can now derive the demand functions for manufactured and service sector goods.

$$Y_m = \left(\frac{P_m}{P}\right)^{-\varepsilon} Y \tag{36}$$

$$Y_s = \left(\frac{P_s}{P}\right)^{-\varepsilon} Y. \tag{37}$$

And the employment in each sector would be

$$N_m = \left(\frac{P_m}{P}\right)^{-\varepsilon} \frac{Y}{A} \tag{38}$$

$$N_s = \left(\frac{P_s}{P}\right)^{-\varepsilon} \frac{Y}{A}. \tag{39}$$

A Firm behavior with inventory

A.1 Introduction

I study a partial equilibrium model of inventory management in infinite horizon discrete time. In this model a manufacturing firm produces output at a remote location. Therefore, it must use its retail units to satisfy consumer demand. The motivation behind this work is to develop a simple model of inventory management that can be combined into general equilibrium models in a tractable manner. There are various version of inventory management models explored in economics and operation research literature, however, to the best of my knowledge, all of them are partial equilibrium models that are not straightforward to incorporate into general equilibrium framework.

It is argued that output price at any time t depends upon the price at $t-1$. The implication of this result is that it is a step toward explaining price stickiness in New-Keyneysian monetary economics models where it assumed to be exogenously given.

A.2 Model

The problem is setup in infinite horizon discrete time. The economic agents have perfect foresight and there is no uncertainty. There is a manufacturing firm that produces a single good as its output using labor as only input. The manufacturing activity takes place at a location different from the location of sale. So, in order to be able to sell its product the manufacturer relies on its retail unit that maintains an inventory of the good. The retail unit faces consumer demand and orders inventory at discrete interval of time once the stock becomes too low. For each order it has to pay administrative cost, independent of the quantity ordered, on top of the per unit price. The payment has to be made one period in advance[1].

A.2.1 Manufacturing unit

The manufacturing unit produces a single good as output using a decreasing returns to scale technology using only labor as input. The manufacturing unit has to supply whatever quantity the retail unit orders at discrete time interval. Suppose at time t_i the retail unit orders ξ_i quantity of goods then the manufacturing firm's optimization problem can be stated as follows.

$$\pi_m = \max_{y_t, \ell_t, c_{t_i}} \left\{ \sum_{t=0}^{\infty} \beta^t(-w_t\ell_t) + \sum_{i=1}^{\infty} \beta^{t_i-1} c_{t_i}\xi_i \right\}$$
$$\text{subject to} \quad y_t = A_t \ell_t^{\alpha}.$$

[1]This assumption is needed to simplify the expressions.

where $\alpha \in (0,1)$ [2] and c_{t_i} is the per unit price of good at the manufacturer's location. Market clearing of the output at discrete interval requires

$$\xi_i = \sum_{t=t_{i-1}}^{t_i-1} y_t; \quad \forall i = 1, \ldots,$$

and $t_0 = 0$.

Assumption 1. *Manufacturing unit behaves competitively in input (labor) market.*

Claim 3. *Under Assumption 1, the solution to the firm's optimization problem is characterized by the following equations:*

$$\alpha \beta^{t_i-1} c_{t_i} A_t \ell_t^{\alpha-1} = \beta^t w_t \tag{40}$$

A.2.2 Retail unit

The retail unit begins with some positive stock of goods (x_0) and faces the consumer demand which is assumed to be deterministic. If the stock is too low then it makes an order to the manufacturing unit at the beginning of the period which costs k irrespective of the size of the order, in addition it also pays a per unit cost c_t. Once the order is fulfilled the retailer also incurs a storage cost $(f(x))$ which is increasing in the size of the inventory. The retailers optimization problem can be stated as follows.

$$\pi_r(x_0) = \max_{p_t, \{t_i, \xi_i\}_{i=1}^{\infty}} \left\{ \sum_{t=0}^{\infty} \beta^t [-p_t \Delta x_t - f(x_t)] dt - \sum_{i=1}^{\infty} \beta^{t_i-1}(k + c_{t_i}\xi_i) \right\}$$

The decrease in inventory will be equal to demand rate. The relationship is given by

$$\Delta x_t = -\kappa p_t^{-\epsilon},$$

where $\epsilon > 1$.

[2] Since this is a partial equilibrium analysis, therefore $\alpha = 1$, constant returns to scale technology, will result in firm demanding infinite amount to labor to produce infinite amount of output. In general equilibrium model, this situation is prevented from arising due to supply side constraint.

Now consider the interval $t_i \leq t < t_{i+1}$, the retail unit orders inventory at beginning of t_i will be sold before time t_{i+1}. The firm's reward in this interval will be

$$\sum_{t=t_i}^{t_{i+1}-1} \beta^{(t-t_i)}[-p_t \Delta x_t - f(x_t)] - \sum_{t=t_{i-1}}^{t_i - 1} c_{t_i} y_t - k \tag{41}$$

From Equation (41), observe that it is optimal for the retailer to set its price such that the entire stock of output is sold out immediately, so that the storage cost of inventory is minimized.

Claim 4. *The optimal strategy of the retailer is to set the price p_t such that entire inventory is sold out immediately.*

Proof. Given the rate of demand is $\kappa p_t^{-\epsilon}$, the revenue at any moment as a function of price is given by $\kappa p_t^{1-\epsilon}$ which is a decreasing function of p_t.

Secondly, in general, the storage cost, $(f(x_t))$ will be an increasing function of the amount of inventory being stored.

Lastly, the cost of accumulating inventory at t_i, $\sum_{t_{i-1}}^{t_i - 1} c_{t_i} y_t + k$ has already been incurred at the beginning of t_i and therefore it acts as a fixed quantity in the interval $[t_i, t_{i+1} - 1]$.

Thus retailer optimization will imply that it sets price p_t low enough to sell all the inventory immediately, which simultaneously maximizes the revenue and reduces the storage cost. ■

Hence the retail unit's maximization problem at time t can be written as

$$-p_t \Delta x_t - f(x_t) - c_t \xi_t - k \tag{42}$$

The above equation is further simplified in the following manner. The rate of decrease in inventory is

$$\Delta x_t = -\kappa p_t^{-\epsilon}$$

Recall that at the beginning of every period t, the retail unit orders inventory ξ_t which is sold out by the end of period t, before it begins the new cycle. Hence,

$$\xi_t = \kappa p_t^{-\epsilon}$$

Assumption 2. *The functional form of $f(x)$ is given by $f(x) = \sigma p_t x$, where σ is a constant parameter of the model.*

Now the retailers optimization problem at t becomes

$$\left\{\kappa p_t^{1-\epsilon} - (\sigma p_t + c_t)\kappa p_t^{-\epsilon}\right\} - k.$$

Claim 5. *The profit maximizing price is $p_t = \frac{\epsilon}{\epsilon - 1}(\sigma p_t + c_t)$. And the quantity of inventory ordered is $\kappa \left(\frac{\epsilon}{\epsilon-1}(\sigma p_t + c_t)\right)^{-\epsilon}$.*

From Equation (40), the supply at the beginning of t is $A_{t-1}\left(\frac{\alpha\beta A_{t-1}c_t}{w_{t-1}}\right)^{\frac{\alpha}{1-\alpha}}$. The per unit ordering cost c_t can be obtained by equating the demand rate and supply rate. That is, c_t is determined by

$$\kappa \left(\frac{\epsilon}{\epsilon-1}(\sigma p_t + c_t)\right)^{-\epsilon} = A_{t-1}\left(\frac{\alpha\beta A_{t-1}c_t}{w_{t-1}}\right)^{\frac{\alpha}{1-\alpha}}. \tag{43}$$

In the special case of no storage cost, $\sigma = 0$, the expression for c_t can be determined analytically,

$$c_t^{\frac{\alpha}{1-\alpha}+\epsilon} = \left(\frac{\kappa\left(\frac{\epsilon}{\epsilon-1}\right)^{-\epsilon}}{(\alpha\beta)^{\frac{\alpha}{1-\alpha}}}\right)\cdot\left(\frac{w_{t-1}^{\alpha}}{A_{t-1}}\right)^{\frac{1}{1-\alpha}}.$$

Through Equation (43), c_t is a function of prices in the previous instant w_{t-dt}, and therefore through its dependence on c_t, p_t is a function of previous period price.

A model with no inventory

For comparison, consider a model of a monopolist firm producing the output with same technology, $y_t = A_t\ell_t^{\alpha}$, and facing the same demand rate $y_t = \kappa p_t^{-\epsilon}$, where ℓ_t is the labor input employed, A_t is the productivity, and p_t is the price of the final output.

The profit maximization of this firm can be stated as follows:

$$\max_{p_t} p_t y_t - w_t \ell_t,$$

$$\text{s.t.} \quad y_t = A_t \ell_t^{\alpha},$$

$$y_t = \kappa p_t^{-\epsilon}.$$

For this model, it can be shown that the firm will optimally set its price to be

$$p_t = \left[\frac{\kappa^{\frac{1-\alpha}{\alpha}\epsilon}}{\alpha(\epsilon - 1)} \cdot \frac{w_t}{A_t^{\frac{1}{\alpha}}} \right]^{\frac{1}{1+\frac{(1-\alpha)\epsilon}{\alpha}}},$$

which is independent of prices in the previous periods.

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